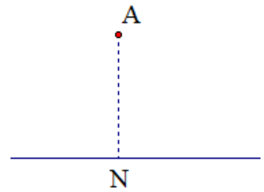
Vectors II – Lines

	Formula	Example
1	**Equation of line** **Vector form** $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$, $\lambda \in \mathbb{R}$ **a** is a fixed point on the line, **b** is direction of line **Parametric form** Example: Given $\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\5\\6\end{pmatrix}$ Replace **r** with $\begin{pmatrix}x\\y\\z\end{pmatrix}$, we have $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\5\\6\end{pmatrix}$, hence $x = 1+4\lambda$, $y = 2+5\lambda$, $z = 3+6\lambda$ **Cartesian form** $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{6} \quad (=\lambda)$	$\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\5\\6\end{pmatrix}, \lambda \in \mathbb{R}$
2	Angle between 2 lines $\cos\theta = \frac{\mathbf{b}_1 \bullet \mathbf{b}_2}{\lvert\mathbf{b}_1\rvert \lvert\mathbf{b}_2\rvert}$, where $\mathbf{b}_1$ and $\mathbf{b}_2$ are the directions of the lines Note: If **acute** angle, then $\cos\theta = \frac{\lvert\mathbf{b}_1 \bullet \mathbf{b}_2\rvert}{\lvert\mathbf{b}_1\rvert \lvert\mathbf{b}_2\rvert}$	$l_1 : \mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\5\\6\end{pmatrix}$ $l_2 : \mathbf{r} = \begin{pmatrix}2\\3\\4\end{pmatrix} + \mu\begin{pmatrix}2\\5\\7\end{pmatrix}$ $\cos\theta = \frac{\begin{pmatrix}4\\5\\6\end{pmatrix} \bullet \begin{pmatrix}2\\5\\7\end{pmatrix}}{\sqrt{77}\sqrt{78}} = \frac{75}{\sqrt{77}\sqrt{78}}$ $\theta = 14.6°$
3	**Intersection of 2 lines** To find the intersection 2 lines, equate the equations of the 2 lines, for example, given $l_1 : \mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\5\\6\end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix}2\\3\\4\end{pmatrix} + \mu\begin{pmatrix}2\\5\\7\end{pmatrix}$, $l_1 = l_2$ $\begin{pmatrix}1+4\lambda\\2+5\lambda\\3+6\lambda\end{pmatrix} = \begin{pmatrix}2+2\mu\\3+5\mu\\4+7\mu\end{pmatrix}$ solve any 2 equations to find the values of λ and μ i.e. $\begin{matrix}4\lambda - 2\mu = 1\\5\lambda - 5\mu = 1\end{matrix}$ to get $\lambda = \frac{3}{10}$ $\mu = \frac{1}{10}$ Then sub. the values of λ and μ into the 3rd equation. If LHS=RHS, then the lines intersect, otherwise the lines do not intersect and they are *skew*. **To find the point of intersection, sub.** λ or μ into the equation of either line.	

4*	<u>**Foot of perpendicular from a point to a line**</u>

Example: Given $A(2,3,7)$ and $l_1 : \mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\5\\6\end{pmatrix}$



Step 1: Let N be the foot of perpendicular from A to the line.

Since N lies on the line, let $\overrightarrow{ON} = \begin{pmatrix}1+4\lambda\\2+5\lambda\\3+6\lambda\end{pmatrix}$

Step 2: Find $\overrightarrow{AN}$

$$\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA}$$

$$= \begin{pmatrix}1+4\lambda\\2+5\lambda\\3+6\lambda\end{pmatrix} - \begin{pmatrix}2\\3\\7\end{pmatrix} = \begin{pmatrix}-1+4\lambda\\-1+5\lambda\\-4+6\lambda\end{pmatrix}$$

Step 3: Since AN is perpendicular to line l_1,

$$\overrightarrow{AN} \bullet \begin{pmatrix}4\\5\\6\end{pmatrix} = 0$$

$\begin{pmatrix}-1+4\lambda\\-1+5\lambda\\-4+6\lambda\end{pmatrix} \bullet \begin{pmatrix}4\\5\\6\end{pmatrix} = 0$, solve the equation to find λ.

Step 4: Sub. λ into step 1 to get $\overrightarrow{ON}$.

If you need to find the perpendicular distance, then find $\left|\overrightarrow{AN}\right|$.