VECTORS 1 Summary

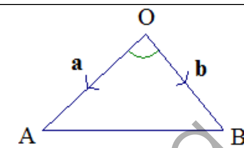
This list is not exhaustive. It is a summary of the most important and commonly used formulas for vectors 1.

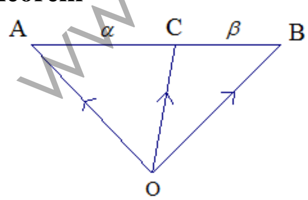
Magnitude of vector

Given $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

<table>
<tr><th></th><th>Formula</th><th>Example</th></tr>
<tr><td>1</td><td>Dot product(Scalar product)
$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

Note: $\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{a}$</td><td>$\mathbf{a} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}4\\5\\6\end{pmatrix}$
$\mathbf{a} \bullet \mathbf{b} = \begin{pmatrix}1\\2\\3\end{pmatrix} \bullet \begin{pmatrix}4\\5\\6\end{pmatrix} = 4 + 10 + 18 = 32$</td></tr>
<tr><td>2</td><td>Angle between 2 vectors a and b
$\cos\theta = \dfrac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

Note:
1. Both vectors a and b must converge/diverge from the same point.
2. If <u>acute</u> angle, then $\cos\theta = \dfrac{|\mathbf{a} \bullet \mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}$</td><td>

$\cos A\hat{O}B = \dfrac{\begin{pmatrix}1\\2\\3\end{pmatrix} \bullet \begin{pmatrix}4\\5\\6\end{pmatrix}}{\left|\begin{pmatrix}1\\2\\3\end{pmatrix}\right|\left|\begin{pmatrix}4\\5\\6\end{pmatrix}\right|} = \dfrac{32}{\sqrt{14}\sqrt{77}}$
$A\hat{O}B = 12.9°$</td></tr>
<tr><td>3</td><td>(a) If 2 vectors a and b are <u>perpendicular</u>,
$\mathbf{a} \bullet \mathbf{b} = 0$
(b) If 2 vectors are <u>parallel</u>,
a = kb, where k is a constant.
(c) If 3 points A, B and C are collinear,
$\vec{AB} = k\,\vec{AC}$, where k is a constant and B is a common point</td><td></td></tr>
<tr><td>4</td><td>Unit Vector, $\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$</td><td></td></tr>
<tr><td>5</td><td>Ratio Theorem


Given that the point C divides AB in the ratio $\alpha : \beta$,
$\vec{OC} = \dfrac{\alpha\vec{OB} + \beta\vec{OA}}{\alpha + \beta}$.
If C is the midpoint of AB, then
$\vec{OC} = \dfrac{\vec{OA} + \vec{OB}}{2}$.</td><td></td></tr>
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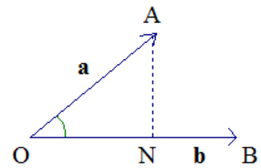
Projection of **a** on **b**:



Length of projection *ON*, of **a** on **b** $= \left|\mathbf{a}\bullet\hat{\mathbf{b}}\right| = \left|\dfrac{\mathbf{a}\bullet\mathbf{b}}{\mathbf{b}}\right|$

Projection vector, $\vec{ON} = \left|\mathbf{a}\bullet\hat{\mathbf{b}}\right|\hat{\mathbf{b}}$

Perpendicular distance from *A* to *OB*,

$$\vec{AN} = \left|\mathbf{a}\times\hat{\mathbf{b}}\right|$$

<u>Resolving a vector along parallel and perpendicular directions</u>

Vector **a** resolved along a direction parallel to *OB* is

$\vec{ON} = \left|\mathbf{a}\bullet\hat{\mathbf{b}}\right|\hat{\mathbf{b}}$

Vector **a** resolved in a direction perpendicular to *OB* is $\mathbf{a} - \left|\mathbf{a}\bullet\hat{\mathbf{b}}\right|\hat{\mathbf{b}}$

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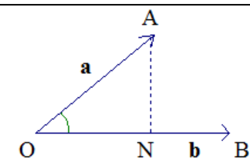

$\mathbf{a} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}4\\5\\6\end{pmatrix}$

Length ON$= \dfrac{\begin{pmatrix}1\\2\\3\end{pmatrix}\bullet\begin{pmatrix}4\\5\\6\end{pmatrix}}{\left|\begin{pmatrix}4\\5\\6\end{pmatrix}\right|} = \dfrac{32}{\sqrt{77}}$ units

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Cross product(Vector product)

$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta$

Note: (i) $\mathbf{a}\times\mathbf{b}$ gives a vector perpendicular to both **a** and **b**.

(ii) $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$

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$\mathbf{a} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}4\\5\\6\end{pmatrix}$

$\mathbf{a}\times\mathbf{b} =$

$\mathrm{i}(12-15) - \mathrm{j}(6-12) + \mathrm{k}(5-8) = \begin{pmatrix}-3\\6\\-3\end{pmatrix}$

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Area of triangle$= \dfrac{1}{2}\left|\mathbf{a}\times\mathbf{b}\right|$

Area of //gram$= \left|\mathbf{a}\times\mathbf{b}\right|$

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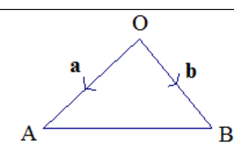

Area of $\Delta OAB = \dfrac{1}{2}\left|\mathbf{a}\times\mathbf{b}\right|$

$= \dfrac{1}{2}\left|\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}4\\5\\6\end{pmatrix}\right| = \dfrac{1}{2}\left|\begin{pmatrix}-3\\6\\-3\end{pmatrix}\right|$

$= \dfrac{1}{2}\sqrt{(-3)^2 + (6)^2 + (-3)^2}$

$= \dfrac{3}{2}\sqrt{6}\text{units}^2$

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<u>Coplanar Vectors</u>

If vectors **a**, **b**, **c** are coplanar, then $\mathbf{a}\times\mathbf{b}$ gives a vector perpendicular to **c**(or in any order), i.e.

$$(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = 0$$

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